MA 125
BUSINESS CALCULUS
FALL 2025

5.3 Consumer Loans and Amortization

Amortization

Spreading out a payment or cost over time.

- Loans.
  In lending, amortization means paying off a loan through regular, scheduled payments that include both the interest (usually the compound interest, but it can be simple interest too) and the original amount.

- Accounting.
  In accounting, amortization refers to spreading out the cost.

Recall: Present Value of an Ordinary Annuity

$$PV = PMT\left[\frac{1-\left(1+\frac{r}{m}\right)^{-n}}{\frac{r}{m}}\right]$$

where

$$\begin{aligned} PV &:= Present\ value \\ PMT &:= Payment\ at\ the\ end\ of\ the\ each\ period \\ r &:= Anual\ interest\ rate \\ m &:= Number\ of\ periods\ per\ year \\ n &:= Total\ number\ of\ periods. \end{aligned}$$

In present value of an ordinary annuity problems, we know PMT and want to find PV. In amortization problems, we know PMT and want to find PV. So,

$$PMT = \frac{PV}{\left[\frac{1-\left(1+\frac{r}{m}\right)^{-n}}{\frac{r}{m}}\right]}.$$

By simplifying, we get

$$\boxed{PMT = \frac{PV \cdot \frac{r}{m}}{1 - \left(1 + \frac{r}{m}\right)^{-n}}} \tag{1}$$

Example 1. Find the monthly payment necessary to amortize a 5% loan of \$150,000 over 10 years. (Round to the nearest cent.)

Example 2. For a student loan of \$20,000 at 3% for 15 years, find the following.

(i). The monthly payment necessary to amortize the loan amount. (Round to the nearest cent as needed.)

(ii). The amount of money saved over the lifetime of the loan if an additional \$200 is added to the monthly payment.

Remaining Balance

Even though equal payments (PMT) are made to amortize a loan, the loan balance does not decrease in equal steps.

The remaining balance B after x number of payments is given by

$$\boxed{B = PMT\left[\frac{1-\left(1+\frac{r}{m}\right)^{-(n-x)}}{\frac{r}{m}}\right]}. \tag{2}$$

Here,

$$\begin{aligned} PMT &:= \textit{Payment at the end of the each period} \\ r &:= \textit{Anual interest rate} \\ m &:= \textit{Number of periods per year} \\ n &:= \textit{Total number of periods.} \end{aligned}$$

Example 3. Find the monthly payment and estimate the remaining balance. Assume interest is on the unpaid balance.

Thirty-year mortgage for $\$250,000$ at 3.66%; remaining balance after 12 years.

(i). The monthly payment is .... (Round to the nearest cent.)

(ii). The remaining balance is .... (Round to the nearest dollar.)

Amortization Schedules

To determine the exact remaining balance after each loan payment, financial institutes normally use an **amortization schedule** which lists how much of each payment is interest, how much goes to reduce the balance, and how much is still owed after each payment.

Example 4. Using the following amortization table, find the following.

(i). How much of the 7^{th} payment is interest? ....

(ii). How much interest is paid in the first three months of the loan? ....

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	–	–	–	$1000.00
1	$88.85	$10.00	$78.85	$921.15
2	$88.85	$9.21	$79.64	$841.51
3	$88.85	$8.42	$80.43	$761.08
4	$88.85	$7.61	$81.24	$679.84
5	$88.85	$6.80	$82.05	$597.79
6	$88.85	$5.98	$82.87	$514.92
7	$88.85	$5.15	$83.70	$431.22
8	$88.85	$4.31	$84.54	$346.68
9	$88.85	$3.47	$85.38	$261.30
10	$88.85	$2.61	$86.24	$175.06
11	$88.85	$1.75	$87.10	$87.96
12	$88.84	$0.88	$87.96	$0.00

Example 5. Mike buys a car costing $5500. He agrees to make payments at the end of each month for 6 years at a compound interest rate of 8.4%. Find the following.

(i). The amount of each payment.

(ii). The total amount of interest Mike will pay.

Present Value of An Annuity Due

Recall: The Present Value of An Annuity Due is given by

$$PV = PMT\left[\frac{1-\left(1+\frac{r}{m}\right)^{-(n-1)}}{\frac{r}{m}}\right] + PMT$$

where

$$\begin{aligned}
PV &:= \textit{Present value}\\
PMT &:= \textit{Payment at the } \textbf{begining} \textit{ of the each period}\\
r &:= \textit{Anual interest rate}\\
m &:= \textit{Number of periods per year}\\
n &:= \textit{Total number of periods.}
\end{aligned}$$

Example 6. A lottery company's handbook discusses the options for how to receive the winnings for a \$4 million ticket.

Option 1: Take 20 annual payments of \$200,000 (that is, \$4 million divided into 20 equal payments).

Option 2: Take a *lump-sum payment* (which is often called *the cash value*).

If the Lottery Commission can earn 5% annual interest, what is the cash value?

Solution. Notice that

$$PMT = \$200,000;\ r = 5\% = 0.05;\ n = 20.$$

Thus,

$$PV = PMT\left[\frac{1-\left(1+\frac{r}{m}\right)^{-(n-1)}}{\frac{r}{m}}\right] + PMT$$

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Interpretation:
This means that receiving \$200,000 at the beginning of each year for 20 years is financially equivalent to having

\$.. today,

if the money earns 5% interest per year.